MAST31213 Complexity theory
Master's Programme in Mathematics and Statistics
Fall 2019
Exercise set 8 (for 1.11.)

Read chapters 2.6 – 2.7 of the book.

Exercise 1. Give a definition of **NEXP** without using nondeterministic Turing machines, analogous to our first definition of the class **NP**, and prove that the definition is equivalent to the one using nondeterminism.

Exercise 2. Show that if $\mathbf{DTIME}(n)=\mathbf{NTIME}(n)$, then $\mathbf{P}=\mathbf{NP}$.

Exercise 3. Show that the following language is undecidable:

$$\{\llcorner M \lrcorner : \ M \text{ is a machine that runs in } 100n^2 + 200 \text{ time}\}.$$

Exercise 4. Show that if $L \in \mathbf{NP} \cap \mathbf{coNP}$ is $\mathbf{NP}$-hard, then $\mathbf{NP} = \mathbf{coNP}$.

Exercise 5. What consequences can we deduce if there are or aren't $\leq_p$-maximal (complete) languages in $\mathbf{NP} \cup \mathbf{coNP}$ or $\mathbf{NP} \cap \mathbf{coNP}$? (Note: some directions are hard open questions of complexity theory, but for which cases can we say something with the current knowledge from the course?)