MAST31213 Complexity theory
Master's Programme in Mathematics and Statistics
Fall 2019
Exercise set 9

Read chapters 3.1–3.2 of the book.

Exercise 1. Show that if f and g are time constructible and such that $f(n) = o(g(n))$ then there is a computable function h such that $f(n) = o(h(n))$ and $h(n) = o(g(n))$ and $h(n)$ is computable in $O(g(n))$-time.

Exercise 2. Show that we can do the interval finding step of the nondeterministic time hierarchy theorem in time $g(n)$, i.e., if $g'(n) = o(g(n))$ and h is defined by

$$h(0) = 1 \qquad h(i+1) = 2^{g'(h(i))}$$

then for any n we can find i such that $h(i) < n \leq h(i+1)$ in time $g(n)$.

Exercise 3. Prove the deterministic time hierarchy theorem: if f and g are time constructible functions satisfying $f(n)logf(n) = o(g(n))$ then $\mathbf{DTIME}(f(n)) \subsetneq \mathbf{DTIME}(g(n))$.

Exercise 4. Show that $\mathbf{coNP} \subseteq \mathbf{EXP} \subseteq \mathbf{coNEXP}$. What do we know about the inclusions?

Exercise 5. Say that a class C_1 is *superior to* a class C_2 if there is a machine M_1 in class C_1 such that for every machine M_2 in class C_2 and every large enough n, there is an input of size between n and n^2 on which M_1 and M_2 answer differently.

(a) Is $\mathbf{DTIME}(n^{1.1})$ superior to $\mathbf{DTIME}(n)$?
(b) Why does our proof of the Nondeterministic Time Hierarchy Theorem not prove that $\mathbf{NTIME}(n^{1.1})$ is superior to $\mathbf{NTIME}(n)$?