MAST31213 Complexity theory
Master's Programme in Mathematics and Statistics
Fall 2019
Exercise set 11

Read chapters 3.4 and 4.1 – 4.2 of the book.

Exercise 1. (Number 6 from last week) Show that there is an oracle A and a language $L \in \mathbf{NP}^A$ such that L is not polynomial-time reducible to `3SAT` even when the machine computing the reduction is allowed access to A. Hint: Look at the B-case of the B-G-S theorem.

Exercise 2. Show that $\mathbf{SPACE}(n) \neq \mathbf{NP}$. Note that we don't know if either is a subset of the other.

Exercise 3. Show that $\mathbf{NP} \subseteq \mathbf{PSPACE}$.

Exercise 4. The configurations we looked at in class assume that the machine never looks at the input tape beyond the input (or input+1 to detect its ending). Show that we can make this assumption without loss of generality, i.e., if S is space-constructible and M is a Turing machine running in space $S(n)$, then there is another Turing machine M' (with k extra work tapes for some constant k depending on M) running in space $S(n)$ such that it never moves more than one step beyond the input on the input tape. Hint: show that going beyond the input can be simulated with a clock.

Exercise 5. Prove the *Space Hierarchy Theorem*: If f, g are space-constructible functions satisfying $f(n) = o(g(n))$, then

$$\mathbf{SPACE}(f(n)) \subsetneq \mathbf{SPACE}(g(n)).$$