MAST31213 Complexity theory
Master's Programme in Mathematics and Statistics
Fall 2019
Exercise set 12

Read chapters 4.2 – 4.3 of the book.

Exercise 1. Prove that every language L that is not the empty set or $\{0,1\}^*$ is **NL**-hard under polynomial-time Karp reductions. (Why do we need to exclude the empty set and $\{0,1\}^*$?)

Exercise 2. Define a function $f : \{0,1\}^* \to \{0,1\}^*$ to be write-once logspace computable if it can be computed by an $O(\log n)$-space Turing machine M whose output tape is 'write-once' in the sense that, in each step M can either keep its head in the same position on that tape or write to it a symbol and move one location to the right. The used cells of the output tape are not counted against M's space bound.

Prove that f is write-once logspace computable if and only if it is implicitly logspace computable.

Exercise 3. Show that `TQBF` is complete for **PSPACE** also under logspace reductions.

Exercise 4. Suppose we define **NP**-completeness using logspace reductions instead of polynomial-time reductions. Show that `SAT` and `3SAT` continue to be **NP**-complete under this new definition. Conclude that `SAT` $\in$ **L** iff **NP**=**L**. Hint: proof of the Cook-Levin Theorem

Exercise 5.

(a) Prove that the read-once certificate definition of **NL** is, indeed, equivalent to the definition using nondeterministic Turing machines.
(b) Prove that in the certificate definition of **NL** if we allow the verifier machine to move its head back and forth on the certificate, then the class being defined changes to **NP**.