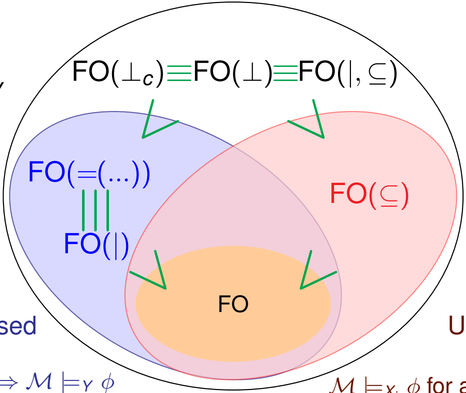
Dependence Logic

December 12 (Thursday), 2019

[Summary]

Basics

$=\!(\vec{x}, y) \equiv y \perp_{\vec{x}} y$



Downwards closed

$\mathcal{M} \models_X \phi$ & $Y \subseteq X \Longrightarrow \mathcal{M} \models_Y \phi$

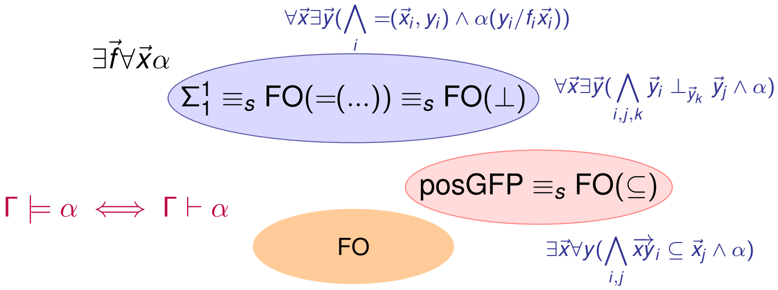
Union closed

$\mathcal{M} \models_{X_i} \phi$ for all $i \in I \Longrightarrow \mathcal{M} \models_{\bigcup_{i \in I} X_i} \phi$

Flat

$\mathcal{M} \models_X \phi \iff \mathcal{M} \models_{\{s\}} \phi$ for all $s \in X$

Expressive power, normal forms, and partial axiomatizations



- Thm. For any τ-sentence ϕ in FO(D), there exists a τ-sentence ψ in L s.t.
$$\mathcal{M} \models_{\{\emptyset\}} \phi \iff \mathcal{M} \models \psi.$$
And vice versa.
- Thm. For any τ-formula $\phi(\vec{x})$ in FO(D), there exists a $\tau(R)$-sentence $\psi(R)$ in Σ^1_1 s.t.
$$\mathcal{M} \models_X \phi(\vec{x}) \iff (\mathcal{M}, X[\vec{x}]) \models \psi(R). \qquad (*)$$
Conversely, for any $\tau(R)$-sentence $\psi(R)$ in Σ^1_1, there exists a τ-formula $\phi(\vec{x})$ in FO($\perp$) s.t. $(*)$ holds for $X \neq \emptyset$.
If $\psi(R)$ is downwards monotone w.r.t. R, then ϕ can be found in FO$(=\!(\ldots))$.

Variants and extensions

$$\mathrm{SO} \equiv_s \mathrm{FO}(=(\ldots), \sim) \equiv_s \mathrm{FO}(=(\ldots), \dot{\sim}) \equiv_s \mathrm{FO}(=(\ldots), \rightarrow) \equiv_s \mathrm{FO}(\subseteq, \hookrightarrow)$$

$$\Sigma^1_1 \equiv_s \mathrm{FO}(=(\ldots)) \equiv_s \mathrm{FO}(\perp)$$

$$\mathrm{posGFP} \equiv_s \mathrm{FO}(\subseteq)$$

$$\mathrm{FO} \equiv_s \mathrm{FO}(\mathcal{D}^{\bar{\uparrow}_0}) \equiv_s \mathrm{FO}(\neq(\ldots)) \equiv_s \mathrm{FO}(\not\subseteq) \equiv_s \mathrm{FO}(\not\perp) \equiv_s \mathrm{FO}(\sim)$$

$\mathcal{D}^{\bar{\uparrow}_0}$ is upwards closed and first-order definable