COMPLEX ANALYSIS I
2020

1. Homework
23.1.2020

1.1. **Homework.** Show that

$$\overline{zw} = \bar{z}\bar{w} \text{ for all } z\,, w \in \mathbb{C}\,.$$

1.2. **Homework.** Find the real and imaginary parts of the complex numbers

$$\frac{4}{(3-i)^2}\,, \qquad \frac{1-2i}{(3+4i)^3} \qquad \left(\frac{i-1}{i+1}\right)^{11}\,.$$

1.3. **Homework.** Let $z \in \mathbb{C}\backslash\{0\}$. Show that

$$z + \frac{1}{z} \in \mathbb{R} \text{ if and only if } \operatorname{Im} z = 0 \text{ or } |z| = 1\,.$$

1.4. **Homework.** (De Moivre's Theorem). Let $\alpha \in \mathbb{R}$. Prove that

$$(\cos\alpha + i\sin\alpha)^n = \cos n\alpha + i\sin n\alpha$$

for all $n \in \mathbb{Z}$.

1.5. **Homework.** Let $z = \sqrt{3} - i$. Find the magnitude of z and the arguments $\operatorname{Arg}_{(-\pi,\pi]}$ and $\operatorname{Arg}_{(0,2\pi]}$ of z. Find the modulus and argument $\operatorname{Arg}_{(0,2\pi]}$ of the number $(\sqrt{3}-i)^5$.

1.6. **Homework.** (An optional extra problem.)
The mapping $z \mapsto \bar{z}$ is a reflection across the real axis in the plane. Let L be a line passing through the origin and forming an angle α with counterclockwise orientation between L and the positive real axis. Find a mapping which gives a reflection across L.