COMPLEX ANALYSIS I
2020

2. Homework
30.1.2020

2.1. **Homework.** (1) Express $\cos 5\alpha$ by using a linear combination of terms $\cos^k \alpha$, $k \geq 1$.
(2) Solve the equation $z^6 + 1 = \sqrt{3}i$. Draw a picture which shows the solutions in the complex plane.

2.2. **Homework.** Let $f : \mathbb{C}\backslash\{-i, i\} \to \mathbb{C}$ be a function defined by

$$f(z) = \left(\frac{z^2 - 1}{z^2 + 1}\right)^9 .$$

When is the function f analytic? Calculate the derivative $f'(z)$.

2.3. **Homework.** (1) When is the function

$$f(z) = \frac{1}{(z + 1/z)^3}$$

analytic? Find the derivative of f.
(2) Consider the function defined by

$$f(x + iy) = \sqrt{|x||y|}\,, \text{ whenever } x, y \in \mathbb{R}\,.$$

Is the function f complex differentiable at the origin?

2.4. **Homework.** Let $x \in \mathbb{R}$. Consider complex valued functions f and g defined in the disc $\mathbb{D}(x, 10)$ by

$$g(z) = \overline{f(\bar{z})}\,.$$

Prove that g is analytic in $\mathbb{D}(x, 10)$ if and only if f is analytic in $\mathbb{D}(x, 10)$.

2.5. **Homework.** Find the set where the function

$$f(z) = \frac{z}{1 + |z|}\,.$$

is complex differentiable.