Problem set 3, Advanced risk theory, 7.2.2020

Exercise 3.1. Consider the Sparre-Andersen model of Section 2.4. Suppose the variable V has an exponential distribution with parameter $\lambda > 0$. That is, the process $\{N(t) \mid t \geq 0\}$ is a Poisson process with intensity λ. Let c_1 be the cumulant generating function of $Y^c(1)$. Assume further that equation $c_1(s) = 0$ has a unique positive root at $s = R$. Let $x > 0$ be fixed. Suppose $c_1'(t) = 1/x$ holds for some $t > R$. Suppose $\Delta > 0$ and set

$$T_\Delta = \inf\{t \in \{\Delta, 2\Delta, 3\Delta, \ldots\} \mid Y^c(t) > U_0\}.$$

That is, T_Δ is the ruin time when the state of the company is only observed at times $\Delta, 2\Delta, 3\Delta, \ldots$.

Show that

$$\mathbb{P}(T_\Delta \leq xU_0) \leq e^{-xc_1^*(1/x)U_0}$$

for all $U_0 > 0$.

Exercise 3.2 (Exercise 3.1 continues)**.** Show that

$$\lim_{U_0 \to \infty} \frac{\log \mathbb{P}(T^c \leq xU_0)}{U_0} = -xc_1^*(1/x).$$

Exercise 3.3. Consider the setting and notations of Example 2.3.

Use the calculations of the example with small changes to show that $Y_{T(U_0)} - U_0$ has an exponential distribution with parameter $\rho - R$ when ξ has the conjugate distribution with parameter R.

Exercise 3.4. Choose an explicit distribution for ξ in Theorem 2.3 so that the distribution satisfies the assumptions of the theorem. Choose $a > \mu$ and calculate the coefficients J_n. Hint: You can choose, for example, exponential distribution.

Exercise 3.5. Explain why the equality holds between Formulas (2.28) and (2.29) in the proof of Theorem 2.3.