COMPLEX ANALYSIS I
2020

3. Homework
6.2.2020

3.1. **Homework.** Suppose that $f : \mathbb{C} \to \mathbb{C}$ is entire. Prove that the function $g(z) = \overline{z}f(z)$ is differentiable at the point z if and only if $f(z) = 0$.

3.2. **Homework.** State and prove the chain rule for analytic functions.

3.3. **Homework.** Let $f : \mathbb{C} \to \mathbb{C}$, be a function such that

$$f(z) = \frac{xy^2(x+iy)}{x^2+y^4}\,, \text{ if } z \neq 0\,, \text{ where } z = x+iy\,, x \in \mathbb{R}\,, y \in \mathbb{R}\,,$$

and $f(0) = 0$. Is the function f complex differentiable at the origin? Find the points where f is complex differentiable. Is the function f analytic anywhere?

3.4. **Homework.** Suppose that $f : \mathbb{C} \to \mathbb{C}$, $f(z) = u(x, y) + iv(x, y)$, is entire and that

$$2u(x, y) + v(x, y) = 5 \text{ for all } z = x + iy \in \mathbb{C}.$$

Show that f is constant.

3.5. **Homework.** Suppose that f is analytic in an open disc $\mathbb{D}(z_0, r)$. Prove that in any one of the following cases:

(1) $\mathrm{Re}(f)$ is constant;
(2) $\mathrm{Im}(f)$ is constant;
(3) $|f|$ is constant;

one can conclude that f is constant.