COMPLEX ANALYSIS I
2020

4. Homework
13.2.2020

4.1. **Homework.** Show that in polar coordinates, the Cauchy-Riemann equations take the form

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \text{ and } \frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}\,.$$

4.2. **Homework.** Define a function $f : \mathbb{C} \to \mathbb{C}$ by

$$f(z) = \begin{cases} \exp(-z^{-4})\,, & \text{if } z \neq 0\,, \\ 0\,, & \text{if } z = 0\,. \end{cases}$$

Show that the Cauchy-Riemann equations are valid at every point of the complex plane, but f is not an entire function.

4.3. **Homework.** For each of the following power series, calculate the radius of convergence.

$$(a) \quad \sum_{n=0}^{\infty} z^n\,, \qquad (b) \quad \sum_{n=1}^{\infty} n z^{n-1}\,, \qquad (c) \quad \sum_{n=1}^{\infty} n^2 z^{n-1}\,.$$

Find the corresponding sums.

4.4. **Homework.** (1) Suppose that $\sum a_n z^n$ has the radius of covergence R. Find the radius of convergence of $\sum a_n z^{2n}$.
(2) Discuss the convergence of the series

$$\sum_{n=0}^{\infty} 2^n |z|^{n^2}\,.$$

4.5. **Homework.** Let (a_n) be a sequence of complex numbers such that $\sum |a_n| < \infty$ and $\sum n|a_n| = \infty$. Prove that the radius of convergence of $\sum a_n z^n$ is 1.