COMPLEX ANALYSIS I
2020

5. Homework
20.2.2020

5.1. **Homework.** Suppose that $\alpha \in \mathbb{R}$, $a \in \mathbb{R}$ such that $|a| < 1$. Prove that

$$1 + a\cos\alpha + a^2\cos 2\alpha + a^3\cos 3\alpha + \cdots = \frac{1 - a\cos\alpha}{1 - 2a\cos\alpha + a^2}$$

$$a\sin\alpha + a^2\sin 2\alpha + a^3\sin 3\alpha + \cdots = \frac{a\sin\alpha}{1 - 2a\cos\alpha + a^2}\,.$$

5.2. **Homework.** (1) Find the real and imaginary parts of the complex number $\exp(\exp(z))$.
(2) Suppose that f is entire and there exists a constant C such that $f'(z) = Cf(z)$ for all $z \in \mathbb{C}$. Determine all these functions f.

5.3. **Homework.** Suppose that $z\,, z_1\,, z_2 \in \mathbb{C}$ and $x, y \in \mathbb{R}$. Prove that

$$\sin(z_1 + z_2) = \sin z_1 \cos z_2 + \cos z_1 \sin z_2\,.$$

Prove Osborn's rules

$$\sin iz = i\sinh z \text{ and } \cos iz = \cosh z\,.$$

Prove that

$$\sin(x + iy) = \sin x \cosh y + i\cos x \sinh y\,.$$

5.4. **Homework.** Determine all the complex numbers z for which $\sin z = -2i$.

5.5. **Homework.** (1) Find the branch of the argument which takes values in $(-\pi/2, 3\pi/2]$, and the corresponding branch of the logarithm and i^{th} power of $(-\sqrt{3} - i)$.
(2) Find the branch of the argument which takes values in $(-\pi, \pi]$, and find the corresponding branch of the logarithm and i^{th} power of $(-\sqrt{3} - i)$.
(3) Determine all the values of $1^{\sqrt{2}}$. Find $\mathrm{Re}(i^{-2i})$. Find $|(-i)^{-i}|$.
(4) Find in the upper half-plane the branch of the $(1 - i)^{\text{th}}$ power of z so that the branch maps $i \mapsto \exp(-3\pi(1 + i)/2)$.