COMPLEX ANALYSIS I
2020

6. Homework
27.2.2020

6.1. **Homework.** (1) Let $a, b \in \mathbb{C}$ and define $\gamma : [0,1] \to \mathbb{C}$ by $\gamma(t) = (1-t)a + tb$. Evaluate the integral $\int_\gamma z\, dz$.
(2) Let $b \in \mathbb{C}$ and let r, α be positive real numbers. Define $\gamma : [0, \alpha] \to \mathbb{C}$ by $\gamma(t) = b + r\exp(it)$. Evaluate the integral

$$\int_\gamma \frac{dz}{z-b} \,.$$

(3) Define $\gamma : [0, 2\pi] \to \mathbb{C}$ by $\gamma(t) = b + r\exp(it)$. Let $n \in \mathbb{Z}$. Evaluate the integral

$$\int_\gamma (z-b)^n \, dz \,.$$

6.2. **Homework.** Let $\gamma_m(t) = t + it^m \, , t \in [0,1] \, ,$ where $m \in \mathbb{N}$ is fixed. Evaluate the following integrals

$$\int_{\gamma_m} z \, dz \quad \text{and} \quad \int_{\gamma_m} \overline{z} \, dz \,.$$

6.3. **Homework.** Define $\gamma : [0, \pi] \to \mathbb{C}$, $\gamma(t) = R\exp(it)$ where $R > 3$. Show that

$$\left| \int_\gamma \frac{\exp(3iz)}{(z^2+4)(z^2+9)} \, dz \right| \leq \frac{\pi R}{(R^2-4)(R^2-9)} \,.$$

6.4. **Homework.** Evaluate

$$\int_\gamma |z|^2 \, dz$$

where γ denotes the contour that goes vertically from 0 to i then horizontally from i to $1+i$. Evaluate the integral also when γ denotes the contour that goes horizontally from 0 to 1 and then vertically from 1 to $1+i$. Does the mapping $z \mapsto |z|^2$ have an antiderivative?

6.5. **Homework.** Suppose that f is analytic in the unit disc $\mathbb{D}$, and f' is continuous there. If $\mathrm{Re}(f'(z)) > 0$ for all $z \in \mathbb{D}$, prove that f is one-to-one, that is, f is injective.