COMPLEX ANALYSIS I
2020

7. Homework
12.3.2020

7.1. **Homework.** Define $\gamma_1 : [0,1] \to \mathbb{C}$ by $\gamma_1(t) = (1+t^2)\exp(i3\pi\sqrt{t})$ and $\gamma_2 : [0,1] \to \mathbb{C}$ by $\gamma_2(t) = 1 + \exp(2\pi i t)$. Evaluate the integrals

$$\int_{\gamma_1} \frac{dz}{z^2}\,, \qquad \int_{\gamma_2} \sin(z^9)\,dz\,.$$

7.2. **Homework.** Suppose that f is defined on $\mathbb{C}$ such that $z \mapsto z^2 \sin z$. Find a primitive of f. Evaluate the integral along any smooth curve from 0 to i.

7.3. **Homework.** Let $\eta \in \mathbb{C}$ and $a \in \mathbb{C}$ be constants such that $|\eta| = 1$ and $|a| < 1$. Define a mapping $f : \mathbb{D} \to \mathbb{D}$ by

$$f(z) = \eta \frac{z+a}{1+\bar{a}z}\,, \quad z \in \mathbb{D}\,.$$

Show that

$$\frac{|f'(z)|}{1-|f(z)|^2} = \frac{1}{1-|z|^2}\,, \quad z \in \mathbb{D}.$$

One can define the hyperbolic metric in the unit disc such that the length of a smooth curve γ in this hyperbolic metric is

$$\ell_{hyp}(\gamma) = \int_\gamma \frac{1}{1-|z|^2}|\,dz|\,.$$

Show that $\ell_{hyp}(f \circ \gamma) = \ell_{hyp}(\gamma)$.

7.4. **Homework.** Goursat's theorem for a triangle: Suppose that D is an open set in $\mathbb{C}$, and $T \subset D$ is a triangle whose interior is also contained in D. Show that

$$\int_{\partial T} f(z)\,dz = 0$$

whenever f is analytic in D.

7.5. **Homework.** Suppose that f is continuously complex differentiable on an open disc $\mathbb{D}$, and $T \subset \mathbb{D}$ is a triangle. Apply Green's theorem to show that

$$\int_{\partial T} f(z)\,dz = 0\,.$$